Evaluation of Estimation Models using the Minimum Interval of Equivalence

José Javier Dolado[a,*], Daniel Rodriguez[b], Mark Harman[c],
William B. Langdon[c], Federica Sarro[c]

[a] *Facultad de Informática, UPV/EHU, Univ. of the Basque Country, Spain.*
[b] *Dept. of Computer Science, Universidad de Alcalá, 28871, Spain.*
[c] *CREST, University College London, WC1E 6BT, UK.*

Abstract

This article proposes a new measure to compare soft computing methods for software estimation. This new measure is based on the concepts of Equivalence Hypothesis Testing (EHT). Using the ideas of EHT, a dimensionless measure is defined using the Minimum Interval of Equivalence and a random estimation. The dimensionless nature of the metric allows us to compare methods independently of the data samples used.

The motivation of the current proposal comes from the biases that other criteria show when applied to the comparison of software estimation methods. In this work, the level of error for comparing the equivalence of methods is set using EHT. Several soft computing methods are compared, including genetic programming, neural networks, regression and model trees, linear regression (ordinary and least mean squares) and instance-based methods. The experimental work has been performed on several publicly available datasets (CSC, China, ISBSG, Maxwell, Desharnais, Atkinson and Telecom1)

Given a dataset and an estimation method we compute the upper point of Minimum Interval of Equivalence, MIEu, on the confidence intervals of the errors. Afterwards, the new measure, MIEratio, is calculated as the relative

[☆] Replication package available at https://github.com/danrodgar/mieratio.
[*] Corresponding author

Email addresses: javier.dolado@ehu.eus (José Javier Dolado), daniel.rodriguezg@uah.es (Daniel Rodriguez), m.harman@cs.ucl.ac.uk (Mark Harman), W.Langdon@cs.ucl.ac.uk (William B. Langdon), f.sarro@ucl.ac.uk (Federica Sarro)

distance of the MIEu to the random estimation.

Finally, the data distributions of the MIEratios are analyzed by means of probability intervals, showing the viability of this approach. In this experimental work, it can be observed an advantage for the genetic programming and linear regression methods by comparing the values of the intervals.

Keywords: software estimations, soft computing, equivalence hypothesis testing, credible intervals, bootstrap

1. Introduction

The search for the best model to estimate software development effort or the code size is a recurring theme in software engineering research. The evaluation and comparison of various estimation models is usually performed using classical hypothesis tests [1, 2] and other tools [3, 4]. Although statistical testing methods have been considered as very powerful techniques in showing that two models are different, the estimates so obtained may not be within a range of any interest. There is a controversy related to the use of the p-values, which have been one of the most used criteria when assessing experimental results [5]. The ban on p-values established by a journal [6] implies that additional criteria must be used when comparing experimental data and methods. One of the most used criterion for comparing software estimation methods is the Mean Magnitude of the Relative Error (MMRE). Despite the fact that it has been proved as inadequate and inconsistent [7, 8], it is still one of the most frequently reported evaluation criterion in the literature. The MMRE is a biased measure that should not be used for comparing models [9].

In this paper, a measure based on the approach of Equivalence Hypothesis Testing (EHT) is proposed. Using the upper point of the Minimum Interval of Equivalence (MIEu) for the absolute error and a random estimation as a reference point, we propose the MIEratio as the relative distance of the MIEu with respect to the random estimation. In this way, those measures will be computed on several publicly available datasets using a variety of estimation methods. At the end of the process, we construct several probability intervals that will allow us the comparison of the methods.

The following steps summarise the evaluation method:

1. Different estimations for each dataset are generated with different estimation methods, varying parameters. A bootstrapped confidence interval of the absolute error of the geometric mean is computed for each dataset, for each estimation method and for each set of parameters.

2. From the confidence intervals generated in the previous step, the one with the upper limit closest to 0 is selected and we take that upper limit point as the "Minimum Interval of Equivalence" (MIEu).

3. A random estimation is computed for each dataset. We assume this is the worst estimation an analyst can make.

4. For each dataset, the values obtained in steps 2 and 3 are used to compute the MIEratio as the measure for assessing the precision of the method.

5. Finally, the MIEratios are grouped by method. The distributions are analyzed and plotted using credible intervals and highest posterior density intervals, taking a Bayesian point of view.

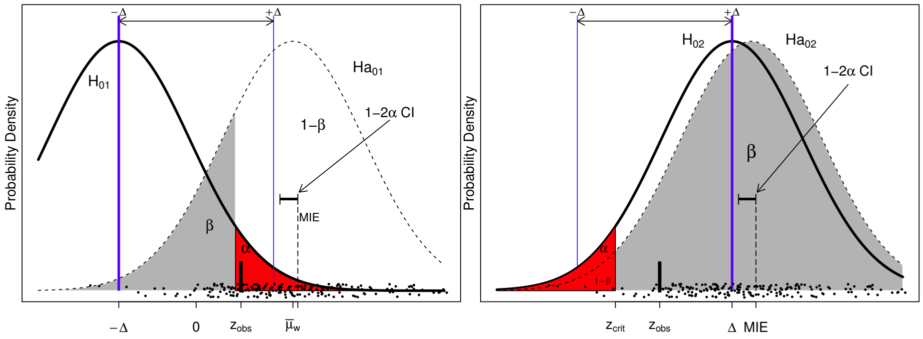
The rest of the article is organized as follows. Section 2 describes the approach followed in step 2, which takes its roots in the *bioequivalence analysis* method used in the medical and pharmacological fields. The elements described form the basis for the rest of the work. Section 3 describes the concepts used in steps 3 and 4 and defines a new measure for classifying methods, the MIEratio (see Section 3.2). Section 4.1 describes the estimation methods and Section 4.2 shows the datasets used. Section 4.3 describes in detail the data analysis procedures and Section 5 presents our results. Next, Section 6 analyzes the data distributions of the MIEratios obtained. Threats to the validity are discussed in Section 7. Finally, Section 8 concludes the paper and highlights future research directions.

2. Equivalence Hypothesis Testing and Confidence Intervals

When making inferences about a population represented by a parameter w, the usual way to proceed is to state a null hypothesis H_0 about the population mean μ_w, $H_0 : \mu_w = \mu_0$, with μ_0 a specified value, and usually $\mu_0 = 0$ when analysing differences. Classical hypothesis testing proceeds by computing a statistic test and examining whether the null hypothesis $H_0 : \mu_w = 0$ can be rejected or not in favour of the alternative hypothesis $H_1 : \mu_w \neq 0$. The statistical tests try to disprove the null hypothesis.

Although classic "Null Hypothesis Significance Test" (NHST) is the standard approach in the software data analysis area, there is an equally valid alternative for the comparison of methods. Under the name of "Equivalence Hypothesis Testing" the null hypothesis is that of "inequality" between the things that we want to compare. This difference is assumed to be larger than a limit Δ. Therefore, the burden of the proof is on the alternative hypothesis of equivalence within the interval $(-\Delta, +\Delta)$. This interval has different names such as "equivalence margin", "irrelevant difference", "margin of interest", "equivalence range", "equivalence limit", "minimal meaningful distance", etc [10].

In EHT, the statistical tests and the confidence intervals are computed to check whether the null hypothesis of inequivalence can be rejected. The main benefit of this approach is that the statistical Type I Error when the null hypothesis is true, commonly named α, is controlled by the analyst, because it has to be predetermined in the null hypothesis. This α is the risk that the analyst is willing to take by wrongly accepting the equivalence of the things compared (i.e., rejecting the assumption of inequivalence). Note that in the NHST the error α has a different interpretation from EHT, i.e., it is the probability of wrongly accepting the difference of the things (rejecting the null difference). Here, the α, or Type I Error, is interpreted in the sense of EHT, i.e., the probability of concluding that the estimates and actual values differ (in absolute terms of the mean) by less than the MIEu when in fact they differ by a value of the MIEu or more. A review of the basic concepts used in EHT can be found in [11, 12, 10, 13].



(a) H$_{01}$ at $-\Delta$.

(b) H$_{02}$ at Δ.

Figure 1: Visualization of the TOST approach. The figure also shows the confidence interval on the mean μ_w outside the interval ($-\Delta$, Δ).

2.1. Confidence intervals and Two One-Sided Tests

There are two common approaches used to carry out the equivalence testing in frequentist statistics: two one-sided tests and confidence interval methods (see for example, Chow and Liu [11, Chap. 4] and Wellek [14, Chap. 3]). In the following, both approaches are outlined.

2.1.1. Two One-Sided Tests

Let us assume that the parameter w has a normal distribution and $\overline{\mu}_w$ is its sample mean. The interval $(-\Delta, +\Delta)$ can be considered as *acceptable* for μ_w, which is also termed as the *irrelevant difference* for μ_w. The rationale for the Two One-Sided Tests (TOST) [15] is based on the fact that an irrelevant difference (or equivalence) within a range $(-\Delta, +\Delta)$ can be established on w by rejecting the two null hypotheses $H_{01} : \mu_w \leq -\Delta$ and $H_{02} : \mu_w \geq \Delta$. If both H$_{01}$ and H$_{02}$ are rejected then the conclusion is that $-\Delta < \mu_w < \Delta$. Fig. 1 shows a hypothetical distribution of values represented by the parameter w, $\overline{\mu}_w$ as the sample mean. For the sake of simplicity, let us assume normal distributions. In Fig. 1(a), we observe that H$_{01}$ is rejected when the one-sided test is performed at $-\Delta$ (with the risk α, Type I Error, set at 0.05) because the observed value from the data, z_{obs}, is within the critical region. Therefore, it can be concluded that the value represented by μ_w is of no practical importance. However, in Fig. 1 (b), when performing a t-test at $+\Delta$, it can be observed that H_{02} is not rejected, therefore equivalence cannot be established. H_{02} is the null hypothesis at Δ and the observed value of the test, z_{obs}, is outside the critical region, so that the null hypothesis of inequivalence cannot be rejected.

2.1.2. Confidence Interval approach

A procedure equivalent to the TOST method is the "confidence interval approach" which basically checks whether the $1 - 2\alpha$ confidence interval of the parameter under study lies within the range $(-\Delta, +\Delta)$ [11, Chap. 4.2].

For illustration purposes, Fig. 1 shows the confidence interval on the mean for the data of a parameter w. The parameter w has practical difference because the confidence interval is outside the interval $(-\Delta, +\Delta)$. Had the confidence interval lied within $(-\Delta, +\Delta)$, the variable would have been considered to represent an irrelevant difference.

Although the size $1 - 2\alpha$ seems a logical consequence of the two one-side tests, Berger and Hsu [16] reported different problems that could arise when generalizing the method to higher dimensions. The origin of the use of a confidence interval for bioequivalence testing dates back to 1972 in an article by Westlake [17]. However, the so-called "Westlake confidence intervals," symmetric around 0, are not in use nowadays due to their larger spans. There were several discussions about the type of interval that should be computed for bioequivalence [18, 19, 15]. The conclusion is that the classical confidence interval with length $1-2\alpha$ for the mean difference should lie within the range $(-\Delta, +\Delta)$ in order to determine equivalence. The confidence interval approach is simple to use and it avoids confusing the interpretation of the *p*-values of the statistical tests, either under NHST or EHT.

The principle of inclusion of the $(1 - 2\alpha)$ confidence interval within the margins is one of the established criteria for showing equivalence and this is the approach adopted in this work. In EHT, the margin limits constitute the range that splits the regions of equivalence-inequivalence.

2.2. Bootstraping Confidence Intervals with the BC_a

An important element of our approach is to compute the confidence intervals and to guarantee that they match the corresponding α-test. Since error distributions do not follow a normal distribution, bootstraping is applied, with the $\mathrm{BC_a}$ (bootstrap bias corrected and accelerated) being the recommended procedure. The statistic bootstraped is the geometric mean, which is more appropriate to log-normal distributions than the standard mean.

The use of the bootstrap method for computing confidence intervals has been applied by several authors in software engineering. A comparison of the application and performance of the different types of confidence intervals in software engineering is described by Lei and Smith [20]. These authors evaluated four bootstrap methods (Normal, Percentile, Student's t and $\mathrm{BC_a}$) and observed that the $\mathrm{BC_a}$ behaved consistently across different software metrics, but the procedure was not free of problems while computing some metrics.

Our present work does not intend to revise the application of bootstrapping to the selected metric used (geometric mean of the absolute error), thus we apply to the standard $\mathrm{BC_a}$ procedure. A brief description of the $\mathrm{BC_a}$ method is explained by Ugarte et al. [21, p. 473]. We report the $\mathrm{BC_a}$ confidence interval following their recommendations [21, Sec. 10.9]. A discussion about the different approaches to the bootstrap of confidence intervals can be found in [22]. The R [23] implementation of the bootstrap, `boot.ci`, guarantees an equi-tailed two-sided nonparametric confidence interval.

2.3. Using EHT for model comparison

Robinson et al. [24, 25] applied equivalence tests for validating prediction models for forest growth measurements in the area of ecological modelling. The authors' starting position (null hypothesis) is that the model is unacceptable and it is the model that has to show some accuracy properties. Although they reported a limited practical utility of their models, the authors showed a good application of equivalence tests as a tool for model validation. As in the current work, they also used bootstraped confidence intervals instead of model-based ones since the assumption of normality could not be guaranteed.

Among other works using EHT for validation purposes in other application areas, Leites et al. [26] used equivalence tests for the assessment of different forest growth models. Equivalence tests were used for validation of the bias, which was defined as the mean of the error of measurements minus predictions. The main benefit reported by these authors is that the "*error of mistakenly validating the model is a Type I Error with a fixed probability.*"

There are recent applications of EHT in the software engineering field. For example, Borg and Pfahl [27] carried out an experiment with eight subjects to compare two requirement traceability tools (Retro and ReqSmile) using EHT. Dolado et al. [13] compared the results of several experimental crossover designs using EHT.

This work is restricted to study to the values of the absolute effort estimation errors. To do so, confidence intervals for the geometric mean of those values are calculated, because we are dealing with the absolute error between observations y_i and predictions $\hat{y}_i$, $|\hat{y}_i - y_i|$. The objective is to find the method that minimizes the geometric mean of the absolute error (gMAR) without any specific interest in any particular estimation in the dataset. We are only interested in setting a limit for equivalence. The margin of equivalence is the smallest value that would include the confidence interval. This value is the largest absolute value of the extreme values of the confidence interval since establishing that value on the margins on $-\Delta$ and $+\Delta$ will make the confidence interval "equivalent" within the limits. The principle of inclusion within a margin will be further described in Section 3.1.

2.4. Other Types of Intervals

There are several works dealing with the evaluation of estimation models using other types of intervals, although those works are not in the line of EHT.

The use of the “Prediction Interval” for estimation purposes has been shown in the works of [28, 29, 30], [31]. The purpose of a prediction interval is different from that of a confidence interval. Heskes [32] shows the differences between confidence intervals and prediction intervals. We refer the reader to [33, Chapter 3] for an explanation with examples about these differences. Mittas et al. [3] have also constructed the tool StatREC that provides different types of graphical intervals and statistical tests.

In Section 6, all data points generated by means of probability intervals are evaluated, which are constructed from a Bayesian perspective.

3. Measures of Accuracy

There are several ways to measure effect size and accuracy of an estimation method. The most common measures used in the software estimation field are the mean of the magnitude of the relative error (MMRE), the median magnitude of the relative error (MdMRE) and the level of predicion at the 25% (LPred(0.25)). A list of the most used measures in other fields can be found in [34, Section 2/5]. They include: root mean squared error (RMSE), mean absolute percentage error (MAPE) and mean absolute scaled error (MASE). The list of measures of accuracy can be extended further. For example, Cumming [35, p. 39] describes a list of potential measures of effect size, such as difference of means, Cohen’s d, the correlation coefficient, etc., each one having specific properties. The selection of one of these measures depends on several factors, such as the field of application, measures previously used in the literature and characteristics of the data.

The comparison of models using machine learning approaches has been a common topic of research since the early works comparing neural networks and genetic programming [36, 37] to the recent systematic review [38]. All comparisons were performed primarily using the MMRE, Level of Prediction at 25% and Median of the MRE. These measures have been used alone or in combination with different statistical tests. A list of problems related to the comparison of estimation methods can be found in [39]. Other issues related to effect size and statistical power in estimation studies have also been reported by [40].

To overcome many of the problems of measuring forecasting accuracy in times series data, Hyndman and Koehler [41] proposed to use the random estimation as a reference point for computing the accuracy of an estimation. Their idea was to scale the error based on the mean absolute error with respect to the naïve (random walk) point.

Shepperd and MacDonell clearly showed the inadequacy of the MMRE for software estimates [7, Table 1]. Based on the idea of Hyndman and Koehler, Shepperd and MacDonell defined a new measure in the field of software estimation called the Standardised Accuracy (SA), which is based on computing a value using both the MAR and the random estimation, $\overline{MAR}_{P_0}$, as a reference point:

$$SA = 1 - \frac{MAR}{\overline{MAR}_{P_0}} \times 100. \tag{1}$$

SA is defined between the values of 0 and 1 (or in percentage between 0% and 100%). Using this measure, the authors concluded that the previous studies about the empirical evaluations of prediction systems are “unsafe.”

We review these concepts already established in software effort estimation field to define a new measure that uses a specific point. That point is the upper limit of a confidence interval of the gMAR. This has the advantage that the evaluation is computed with a preset α level or Type I Error if H_0 is true.

3.1. Minimum Interval of Equivalence

In this Section, the ideas of the EHT for justifying the use of one of the extreme points of a confidence interval as a reference point in a measure of accuracy are described. The EHT procedure uses the margin limits $\pm\Delta$ as the reference points for checking whether or not the confidence interval of the mean lies in $(-\Delta, +\Delta)$. When those margin limits are unavailable or have not been defined, there is the possibility of computing those margins based on a percentage over the mean or over another reference point. There are several works that have used this approach as explained in Section 2.3. In any case, it is possibe to use the equivalence confidence interval itself as a reference point for subsequent uses.

The concept of Minimum Interval of Equivalence (MIE) implies that if there are no clear guidelines for setting the limits of the equivalence interval, one may establish the smallest interval that includes the $1 - 2\alpha$ confidence interval. This is the minimum interval that rejects the null hypothesis of difference or inequivalence, and this is called the MIE. Reporting the MIE for validating a model provides the analyst with the idea of "how close the model was to rejecting the null hypothesis of dissimilarity". Robinson et al. [25, p.912] concluded that the interval of equivalence can be used as a decision-making tool. In this case, the burden of proof is put on the model since the starting assumption is "dissimilarity". The authors suggested to use "the smallest interval of equivalence that would still lead to rejection of the null hypothesis". As an example of application, Miranda et al. compared fire spread algorithms using EHT [42] and reported the minimum interval of equivalence as "the smallest interval that would lead to rejection of the null hypothesis of dissimilarity" (following [24, 25]). Units of MIE were proportions of the mean. Despite the mixed results obtained, the authors found that the information provided by the relative changes in average MIE [42, pp.595] was useful.

The concept of MIE was developed independently by Meyners [43, 10]. He proposed to use "the Least Equivalent Allowable Difference (LEAD)" in equivalence testing for defining the smallest value for which equivalence could be claimed. In this way, given a confidence interval with range (l, u), the largest absolute value of l, u is the LEAD. The positive and negative LEAD values establish an interval in which the confidence interval is included. That interval is the smallest region of equivalence that contains the confidence interval. Meyners [10, Section 10] also found that "the LEAD is particularly useful in situations in which the investigator was unable to choose an equivalence margin."

In the current work, we follow these ideas about finding the confidence intervals and the MIE (or LEAD) will be computed for each parameterised instance of an estimation model. For a given method and dataset we consider as the best confidence interval the one that gives the best MIEu (the upper limit of the interval). Afterwards we compute the corresponding MIEratio, which takes into account the distance with respect to a random estimation. It means that we do not compare the MIEu directly but we use the MIEratio, which is defined in the next section. The use of the MIE concept is valuable since it allows us the comparison of limits irrespectively of any previous definition of the equivalence margin. Also, it is important to remark that the MIEu value depends solely on α, the Type I Error when the null hypothesis is true. Fig. 2 illustrates the concept of the MIE. Given a distribution of residuals or errors – in this case the absolute values are used– a confidence interval for the mean of those values is built. The largest value of the confidence interval sets the limit for equivalence; hence, we can define the upper limit of the Minimum Interval of Equivalence. It is interpreted in the sense of the following: if the limit $+\Delta$ had been set at the MIEu we could have declared "equivalence."

3.2. A New Measure of Accuracy: MIEratio

In this section, a dimensionless measure based on the MIE is defined. The main problem with the previous uses of the MIE is that it has been defined with respect to the sample mean. Using the sample mean as a reference point makes it difficult to compare different models on different datasets. This is especially important in estimation models where the larger the error, the worse it is. Therefore, the concept of "random estimation" is used as part of the new measure for effort estimation in a similar way to the Standardised Accuracy (SA) proposed by Shepperd and MacDonell [7]. The random estimation value is used as a reference point, as originally suggested by Hyndman [41] for measuring the accuracy of time series. The reference point defined by Shepperd and MacDonell for software engineering data is denoted by $\overline{MAR}_{P_0}$.

The $\overline{MAR}_{P_0}$ is defined as the "mean value of a large number runs of random guessing". It equates as predict a $\hat{y}_i$ for the target case i by randomly sampling over all the remaining $n-1$ cases and take $\hat{y}_i = y_r$, where r is drawn randomly

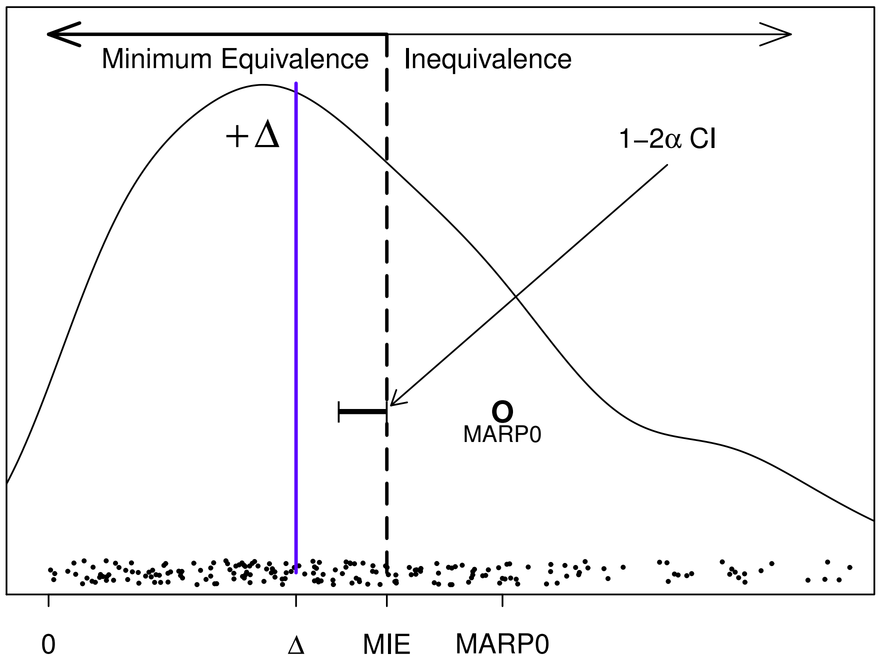


Figure 2: Plot of the $1-2\alpha$ confidence interval of the Absolute Residuals (errors) and the corresponding MIE. The MIEu is the value of the MIE that would lead to rejection of the null hypothesis of dissimilarity.

from $1...n \wedge r \neq i$ (see [7, p. 222]). However, we use "the exact $\overline{MAR}_{P_0}$" proposed by Langdon et al. [44] which consists in iterating over all $n(n-1)$ different combinations of the n data elements and computing the mean of the absolute error. In this way, we avoid "randomness".

Using both the MIEu and the reference point $\overline{MAR}_{P_0}$, the MIEratio is defined as:

$$MIEratio = \frac{MIEu}{(\overline{MAR}_{P_0} - MIEu)} \tag{2}$$

which measures how far the method is from the random estimation, with respect to the minimum range that sets the limit to equivalence. The lower the MIEratio is, the better the estimation method is because it is closer to 0, i.e., further away from $\overline{MAR}_{P_0}$. The $\overline{MAR}_{P_0}$ is constant for each dataset.

While in SA the range of values lies between 0 and 1, in the MIEratio the range of values goes from 0 to $+\infty$. When the MIEratio is equal to 1 there is a situation in which the estimations are equally distant from the perfect estimation (value 0) and from the worst estimation ($\overline{MAR}_{P_0}$). Negative values of the MIEratio are discarded. Values of the MIEratio approach the limit $+\infty$ when the value of the MIEu gets closer to $\overline{MAR}_{P_0}$, which represents bad estimations.

Fig. 3 shows a hypothetical situation in which we have obtained three confidence intervals on the mean of the absolute value of the difference between the actual and the estimated effort. In Fig. 3 it can be observed that the Confidence Interval 1 (computed with a specific α) sets the limit for equivalence and that the MIEratio is defined by the quotient A/B. From the EHT point of view, the A and B regions have an opposite interpretation, equivalence versus inequivalence. The MIEratio is the variable reflecting that relationship.

Due to the fact that the MIEu splits the parameter space into two clear regions with different interpretation (equivalence versus inequivalence, for a specific α), the MIEratio may be used as a measure for comparing estimation methods.

4. Experimental Work

The procedure that we will apply in the next sections is as follows:

1. Take a dataset that provides the *Actual Effort* of a project, split it in three folds, and compute one or several n *estimations* (Estimation 1 to Estimation n for the dataset). The n different estimations are obtained varying different parameters of the estimation method: adjusting constants, adding factors or variables, etc.

2. Compute the *Absolute Error*, which we call it here Absolute Residual (AR), for each estimation. This results in n sets of ARs. Each estimation will have a geometric Mean of the Absolute Residuals (gMAR). Fig. 3 shows the MAR for each confidence interval of the ARs. Note that neither

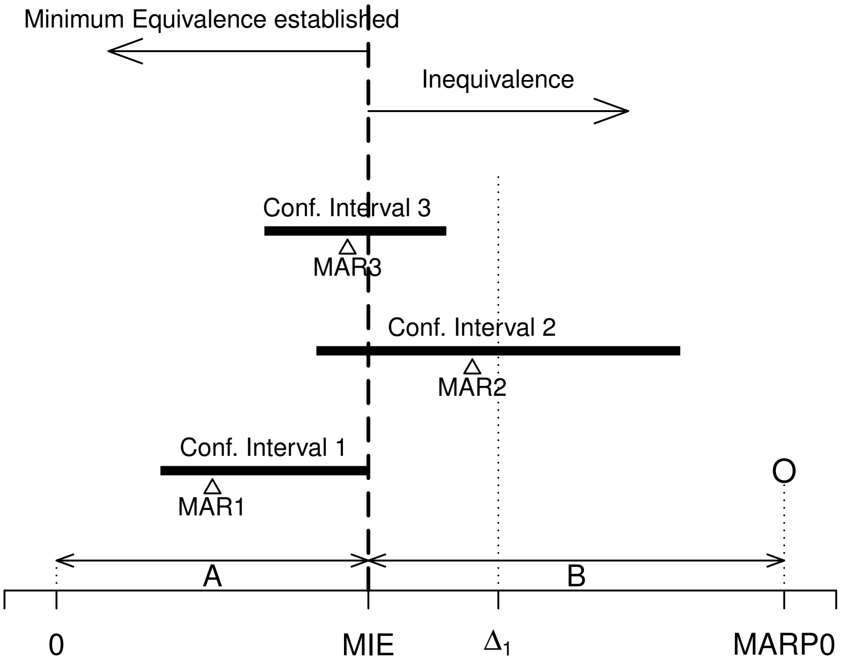


Figure 3: Example of the Minimum Interval of Equivalence and its relationship to $\overline{MAR}_{P_0}$. The MIEratio is defined by the quotient A/B. The x-axis represents the mean of the absolute value of the Actual minus Estimated Effort (MAR).

MARs nor gMARs need to be centred on the confidence intervals. The gMAR is a measure more adequate for skewed distributions.

3. For each set of ARs compute, by bootstraping, the confidence interval on the gMAR with $1-2\alpha$ confidence level. The result of this step is a set of n confidence intervals on the gMAR. The upper endpoint of each confidence interval sets a margin limit for equivalence (to the left of it).

4. From the n confidence intervals obtained in previous step, select the lowest of the upper endpoints of the confidence intervals. That value is the MIEu (or LEAD) for the dataset and estimation method applied.

5. Compute the exact $\overline{MAR}_{P_0}$ for each dataset and fix it for the rest of the

computations.

6. Compute the MIEratio using both the MIEu value and the $\overline{MAR}_{P_0}$ of the previous steps.

7. Repeat steps 1 to 6 for each dataset and for each fold.

8. Sort the results in ascending order of the MIEratio values. The closer the result is to zero, the better the result is.

9. Group the MIEratio values by method, compute the probability intervals and compare their distributions. The closer the result is to zero, the better the result is.

The previous procedure may be repeated for every dataset available and for every estimation method that the analyst wishes to compare.

4.1. Machine Learning software effort estimation methods

The purpose of this work is to show how to compare software estimation methods using the MIEratio, therefore we have generated different software effort estimates using different machine learning algorithms covering most relevant types of techniques usually applied in the software estimation literature. As there can be a high variability in the results depending on the parameters used when applying machine learning methods, we have obtained multiple estimates varying the most important parameters of each technique.

Regression models Regression techniques are the most applied techniques to estimate software effort and in this work we have applied the classical Ordinary Least Squares (OLS) and Least Median Squared (LMS). These techniques fit a multiple linear equation between a dependant variable (*software estimation effort*) with a set of or independent variables that can be numeric or nominal such as the number of function points, team size or the type of development platform.

The aim is to find, using matrix based operations, the slope coefficients of a linear model $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_k x_k + e$ that minimises the square root error. In this work we have used Weka's implementation [45]. Weka is a well-known machine learning software.

Instance based techniques In Instance Based techniques (IB-k), there is no model as such to classify new samples, instead all training instances are stored and the nearest instance(s) is/are retrieved to provide the class or calculate the estimate. In addition to different distance metrics used to compare, other parameters that need to be considered include the number of neighbours (k), use of weights with the attributes, normalisation of the attributes, etc. This technique has been extensively applied by the software engineering community as Case-based Reasoning (CBR) [46].

Genetic Programming Genetic Programming (GP)[47, 48] is a type of evolutionary computation technique in which free form equations evolve to form a symbolic regression equation without assuming any distribution. Several authors have reported on the suitability of using GP in software effort estimation for over a decade [36]. In this work, we have used a canonical implementation based on Weka, which employs trees as a representation and that it is capable of dealing with classification and regression problems.

Neural Networks Neural networks (NN) are one of the classical machine learning techniques applied to software effort estimation. NNs are composed of nodes and arcs organised in layers (usually an input layer, one or two hidden layers and an output layer). Their arcs have weights to control the *propagation* of the *information* that is propagated through the network. There are multiple types of neural networks; the Multilayer Perceptron (MLP) is one of the most popular type in which the *information* is propagated forward from the input layer composed of attributes such as functional size to the output layer nodes (e.g., effort, time, etc.) through one or more hidden layers. Weka's MLP implementation is the

Multilayer Perceptron algorithm in which nodes are sigmoid except with numeric classes in which case output nodes are unthresholded linear units. The loss function during training is the squared-error function.

Regression and model trees Classification and Regression Tress (CART) [49] are binary trees which are induced minimising the subset variation at each branch. In the case of numeric prediction, as it is our case, each leaf represents the average value of the training examples covered at that leaf. We have used Weka's REPTree (Reduction-Error Pruning) [45] algorithm for classification or regression. In the case of regression, variance is used as a splitting criterion and the induced tree can be postpruned to both simplify the tree and to avoid overfitting.

Model trees, originally proposed by Quilan as the M5 algorithm [50], are similar to regression trees but each leaf is composed of a regression equation instead of the average of the observations covered at each leaf, i.e., a linear regression model is induced with the observations of each leaf. Weka's M5P is a improvement of the original M5 algorithm.

In addition to the postpruning process there is also a smoothing process to avoid discontinuities between adjacent linear models. Model tree algorithms should provide higher accuracy than regression trees but they are also more complex.

4.2. Software Engineering Effort Datasets

We apply the methods described in the previous section on seven publicly available datasets. Two of them, China and ISBSG, have a relatively large number of instances. A third one, the CSC dataset is more homogeneous as it is composed of projects that belong to a single company. The Desharnais and Maxwell datasets are well-known and can be found in the PROMISE repositories [1] and in [51], respectively. The two remaining datasets (Atkinson and Telecom1)

[1] http://openscience.us/repo/effort/

are used to compare our results with the results available in the literature, in particular with the results obtained by Shepperd and MacDonell [7].

CSC dataset The CSC data set [52], also known as Kitchenham's data set, is provided by a single company CSC and it is composed of 145 instances. In our study we used the attributes duration, adjusted function points and first estimate as independent variables to estimate the Actual effort (after running an attribute selection algorithm, these were found to be the relevant attributes to deal with effort estimation). This was also the most homogeneous dataset (all data belonged to a single company)

China dataset The China dataset is composed of 499 projects developed in China by various software organisation in multiple domains (cross-company dataset). It has been used in several works, in particular by Menzies et al. [53].

We found some issues with the original dataset and as a consequence we carried out some preprocessing. The *DevType* attribute could not be used as the value of this attribute was zero for all instances. However, we could differentiate between new developments and enhancements (or redevelopments) if the number of function points of the Changed or Deleted field was not zero. We did not use the productivity attributes as they were directly derived from the class attribute (*effort*).

The ISBSG dataset The International Software Benchmarking Standards Group (ISBSG) [54] maintains a software project management repository from multiple organisations.

In this work, we have used ISBSG v10 and it was also necessary to perform some preprocessing in order to apply machine learning techniques, for selecting projects and attributes and for cleaning the data (the preprocessing carried out in this dataset is explained in detail in [55]).

Atkinson and Telecom1 datasets Although the Atkinson and Telecom1 datasets are very small datasets (at least for data mining purposes), these have been extensively used in the past for assessing the estimation-by-analogy method, regression-to-the-mean and other methods.

The Atkinson dataset contains 16 data points relating real-time function points to effort. The Telecom1 dataset contains 18 points relating effort to changes in the configuration management and number of files changed during the development of a software system [46, 56].

The Atkinson dataset is reproduced in full in Appendix B of [56]. The authors report that some cases were removed from the analysis based on the homogeneity of the projects. The 16 points of this dataset have been used for computing the SA (see Table 2 of [7]). The Telecom1 dataset is available in Appendix A of [56] and in Appendix A [46]. It contains 18 data points. This dataset was used for comparing EBA to step-wise regression, among other methods. In these two datasets, we only use the *Actual Effort* and *Estimated Effort* variables for computing the MIE, gMAR, etc.

Maxwell dataset The Maxwell dataset [51] is composed of 63 projects and the following attributes: application size (in Function Points), effort in hours, duration in months. It also provides 21 discrete attributes (applicaytion type, hardware platform, DBMS architecture, user interface, language(s) used, and other 15 attributes using the Likert scale about the development environment characteristics. Although this dataset suffers from the curse of dimensionality as there is a large number of attributes compared with the number of instances, we used all attributes with the exception of the project starting date.

4.3. Data Analysis

All previously described machine learning algorithms are implemented in Weka [45] and they were used to obtain sets of effort estimates per technique (with the exception of the Atkinson and Telecom1 datasets as explained previously). In order to ensure that the same instances were used for training and testing in each technique, we partitioned each dataset into three folds using stratified sampling before applying the machine learning algorithms. Stratified sampling ensures a random sampling following the distribution of the class (effort attribute). We used 2 folds for training (2/3 of the instances) and and 1 fold for testing (1/3 of the instances) and repeat the procecedure three time so that all data points were used for training and testing and to have more points for further analysis. This is in line with the work by Mittas and Angelis [57].

Each machine learning technique was run varying combinations of different parameters to induce a set of different estimates per method (except for the Atkinson and Telecom1 datasets in which the results are those provided in their respective publications). Table 1 shows the parameters modified per method and their respective range. In the case of regression with Ordinary Least Squares (OLS), it is possible to vary the *ridge* parameter (this variation only minimally affects the output) and whether Feature Selection (FS) is used to generate the model. The FS method can be based on the M5 model tree (M5P) which removes attributes until there is no improvement. With Least Median Squared (LMS), it is possible to induce multiple models varying the size of the sample (S) used to generate the regressions. For the GP, we obtained different estimates varying the size of the population (*pop*) and number of generations that we allowed the algorithm to run (*com*). We only allowed arithmetic operators, exponential and logarithms excluding logical (`if`, `and`, `or`) and functions (`min`, `max`). In all cases the fitness function selected was the Root Mean Square Error ($RMSE$).

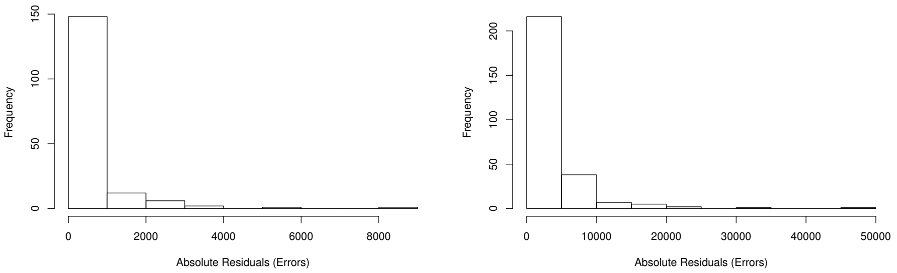
In the case of IBk, there is also a large number of possible parameters but we varied the number of neighbours (k) and the function distance used (Euclidean or Manhattan). For regression and model trees, with the *REPTree* algorithm (regression tree), the parameters were minimum proportion of the variance on all the data that needs to be present at a node for splitting (V), the number of folds (N) which determines the amount of data used for pruning (one fold is used for pruning, the rest for growing the rules) and minimum total weight of the instances in a leaf (M). In the case of M5P (model tree), the parameter modified was the minimum number of instances allowed at a leaf node (M). For

Table 1: Parameters and range of the machine learning algorithms.

Algorithm	*Parameters*
OLR	Ridge: $\{1.0e^{-8}, 0.1\}$
	FS: $\{NoFS, M5P, Greedy\}$
LSM	S: $\{2, 4, 6, 8, 10\}$
GP	Com: $\{100, 200, 400\}$
	Pop: $\{75, 100\}$
IB-k	k: $\{1, 3, 5, 10\}$
	Distance: $\{Euclidean, Manhattan\}$
REPTree	V: $\{1 \cdot 10^{-8}, 0.1, 1\}$
	N: $\{3, 5\}$
	M: $\{2, 5\}$
M5P	M: $\{2, 4, 6, 8\}$
MLP	L: $\{0.25, 0.3, 0.35\}$
	M: $\{0.15, 0.2, 0.25\}$
	N: $\{500, 1000\}$
	H: $\{3, 4\}$

the multilayer perceptron (MLP), we varied the learning rate (L), momentum (M) and training time (N) and the number of hidden layers (H).

For each set of estimations of a method in a dataset, the corresponding confidence intervals are computed and among them, only the best MIEratio was selected. Finally, the best MIEratios are compared. For computing the $\overline{MAR}_{P_0}$ we follow the procedure mentioned in [7]. For obtaining the MIE we build the confidence intervals by bootstrapping (using the R command `boot.ci`). Examples of application of the bootstrap for obtaining confidence intervals can be found in Ugarte et al. [21, Ch. 10]. The problems of building confidence intervals based on percentiles are also stated by Good [58, p. 18]. As a result, we do not report here the confidence intervals based on P_α, (P_0 or $P_{50\%}$); this option could be justified in the work by Shepperd and MacDonell [7] since their histograms showed symmetry.



(a) China dataset using the M5P method.

(b) ISBSG dataset using the IBK method.

Figure 4: Histograms of the absolute errors for two estimations.

5. MIEratios obtained

Here we briefly describe the MIEs that have been obtained on the different datasets applying each estimation method, using $\alpha = 0.05$. Fig. 4 shows two examples of the residuals obtained in the estimations. The x-axis represents effort measured in person-months.

As an example, Fig. 5 shows the plots of the confidence intervals in the validation dataset of China with M5P in fold 1. Four different sets of parameters generate the five different confidence intervals. All intervals are quite similar, meaning that the M5P method behaves uniformly across the parameters. The set of values of “Params 2” gives the lowest MIEu.

GP has shown a behaviour sensitive to the parameters settings. Both length and values of the confidence interval resulted in considerable variations. Fig. 6 shows the plots of the confidence intervals obtained with different sets of parameters for GP in the validation dataset of CSC. Each segment represents a confidence interval obtained by means of bootstrap. The gMAR is also plotted for every confidence interval. The MIEu is obtained with the set of “Params.

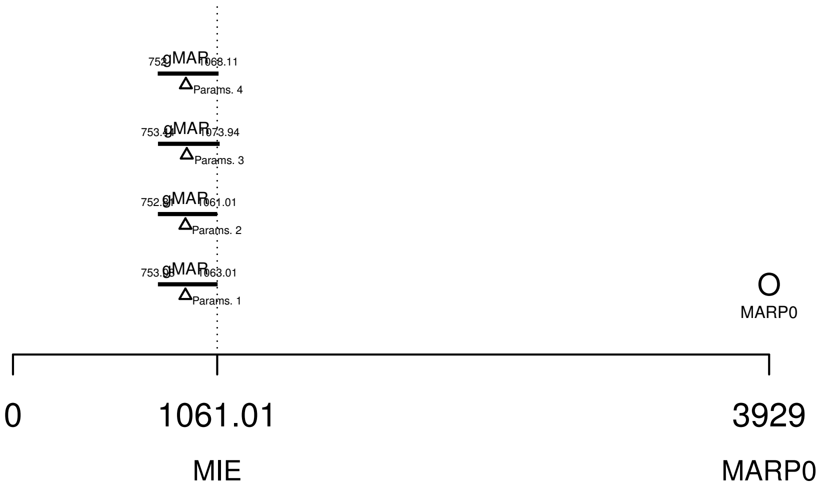


Figure 5: Confidence intervals and the Minimum Intervals of Equivalence for the M5P method applied to the China dataset in fold 1.

5" and it is shown with the vertical dotted line: there is no other smaller value that contains a confidence interval.

The application of neural networks has shown a large variation in the size and location of the confidence intervals depending on the parameters used for adjusting the NNs.

The lowest MIEu values for every dataset and method applied were selected. Visual inspection of the MIEu values does not help to determine which of the methods has performed the best since the $\overline{MAR}_{P_0}$s are different for each dataset. Therefore, the dimensionless MIEratio will help us to establish which of the confidence intervals performs best with respect to the random estimation.

Table 2 shows a subset of the values obtained, using $\alpha = 0.05$, sorted in ascending order by the MIEratio, from the lowest value (best method) up to the highest (worst) value. Each row of the table contains the computed values for a specific estimation method and a dataset.

The list of methods shown in the first column are: genetic programming (GP), IB-k for case-based reasoning, Ordinary Least Squares (LR) and Least Median Square (LMS) for linear regressions, Multilayer Perceptron (MLP) for

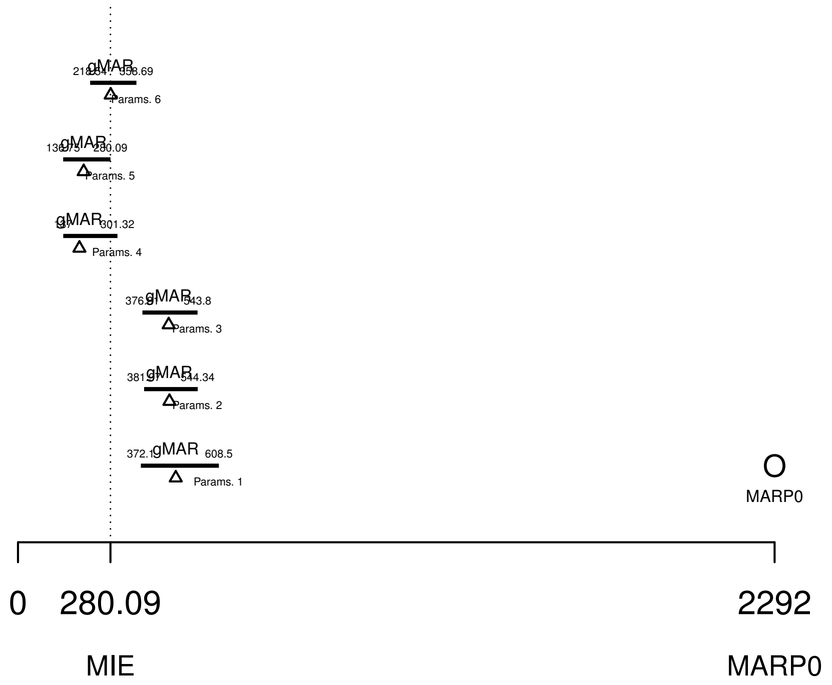


Figure 6: Confidence intervals and the Minimum Intervals of Equivalence for GP and CSC in fold 2.

neural networks, model trees (M5P) and RTree for regression trees. The next columns show the MMRE, MdMRE, Level of prediction and the MIE. The column SA is the "standardised accuracy" of Shepperd and MacDonell. The MIEratio is the criterion used for sorting the table. Therefore, this table compares the most usual measures of accuracy using the MIEratio as the reference.

It can be observed that the order obtained in the last column is not maintained in the rest of the columns. In this situation the main advantage of the MIEratio is that it is computed with a fixed known α, which is essential when comparing estimates obtained from different datasets. The ordering of values that are not matched between the SA and the MIEratio are shown in italics in the column SA.

When establishing an ordering among the estimation methods based on the MIEratio as an accuracy metric, it is observed that the datasets themselves were the main grouping variable in the ordering. This result is in line with some conclusions reported by other authors [57, 59], with respect to the importance of the data itself.

6. Evaluation of the Methods with Probability Intervals

The final question "Which is the best estimation method?" can stated as "which method provides a probability of the MIEratio closer to 0?." A good method should have generated a set of values as close to 0 as possible with respect to the random estimation. The data grouped by method provide a source for making these inferences.

In order to compare the methods, the data of the MIEratios is grouped by method and different probability intervals are constructed for each method. The data that we can use to select an estimation method as the best candidate are the values of the MIEratio obtained in the previous sections. In our case there are 5 datasets (not including the Atkinson and Telecom datasets) multiplied by number of folds (3), that is, a total of 15 data points per method. Had we not splitted the datasets into 3-folds, the number of data points that corresponds to the number of datasets would be very low (5) for the analysis. We consider the data of the MIEratios "observational", hence it is reasonable to generate a probability interval that describes the data distribution.

Probabily intervals are different from confidence intervals. Confidence intervals are constructed from a frequentist point of view. The concept of "probability interval" comes from the area of Bayesian inference. A probability interval, usually called *"credible interval"*, is a range of values in which we can be certain that the parameter falls with a given probability. A 95% credible interval is the range of values in which we are 95% certain that the parameter θ of interest falls (e.g. the MIEratio, a mean or other). On the other hand, a frequentist confidence interval does not convey a probability distribution. A frequentist confidence interval is based on the long-run frequency of the events. Frequentist confidence intervals are computed with the sample data; the procedure guarantees that in the long run a specific percentage of them (e.g. 95%) will contain the true value of the parameter.

Frequentist and Bayesian methods approach the construction of intervals in different ways, because their purpose and aim are different. A Bayesian credible interval is a posterior probability that the parameter lies within the interval constructed. The reader may refer to [60, 61] for a detailed explanation of the differences betweem frequentism and bayesianism.

For our purposes, it suffices to say that a 95% confidence interval is a set of values constructed in such a way that 95% of such intervals will contain the true value of the population and a credible interval is a set of values that represent the probability that the parameter under study will lie within the interval. This probability is computed after the data is observed.

A detailed comparison of the construction between these two types of intervals is described by Cowles [62, Chapter 4] and a description of the construction of probability intervals can be found in Neapolitan's book [63, Section 6.3]. The Bayesian technique is most adequate to make inferences with few data points. It is the interpretation that we are interested in because it is the probability that the true value of the parameter (θ) is in the interval. In some situations both types of intervals (confidence and credible) may provide similar or equal values, but their interpretation is different.

Given the data provided by the MIEratios and grouped by method, we compute three types of Bayesian intervals with the `LaplacesDemon` package for R [64] and other R code.

- A quantile-based interval that computes a 95% probability interval, given the marginal posterior samples of θ. It is simply the 2.5% and 97.5% quantiles of the samples for the distribution. It doesn't take the prior distribution into account.

- The Highest Posterior Density intervals (HPD), recommended for asymmetric distributions. It is the shortest possible interval enclosing $(1-\alpha)\%$ of the distribution. This interval doesn't take the prior distribution into account.

- Credible interval based on Metropolis-Hastings sampling with the use of the non-informative Jeffreys prior for a lognormal model, using the R code of [2]

Table 3 shows the intervals computed, using $\alpha = 0.05$. It can be observed that the best HPD intervals are those of GP. LMS provides good results too. By comparing the values of the intervals in Tables 3 and 5 we see more discriminative power in table 3 due to the scale. Hence, this being the benefit of the current approach.

Table 4 shows the credible intervals for the means of 10 runs, in order to have a perception of how the methods work under different splits of the data. Each run involves variation of all parameters and a different seed for dividing the datasets.

Fig. 7 shows the HPDs for each method. The black areas in the figures represent the 95% probability that the true value of the MIEratio will fall within the interval. The values of the HPDs are those of the second column in Table 3.

7. Threats to Validity

There are some threats to validity that need to be considered in this study. Construct validity is the degree to which the variables used in the study accurately measure the concepts they are supposed to measure. The datasets analyzed here have been extensively used to study effort estimation and other software engineering issues. However, the data is provided "as it is." There is also a risk in the way that the preprocessing of the datasets was performed but it is common to find such approaches in the literature. Furthermore, the aim of this paper is to show how the EHT approach makes it possible to define the MIEratio and to examine what their probability distributions are. Organisations should select and perform the studies and estimations with subsets of the

[2] `http://stats.stackexchange.com/a/33395`

data close to their domain and organisation since there is no clear agreement if cross organisations data can ensure acceptable estimates.

Internal validity is the degree to which conclusions can be drawn. The hold-out approach used could be improved with more complex approaches such as cross validation or “leave one out” validation. A much further range of parameters could have been used to obtain the estimates. However, the computational time makes this difficult in practice. We selected the most important parameters of the algorithms and we used ranges to ensure enough variability of the estimates.

8. Conclusions

This work has shown an analysis of the predictive capabilities of different estimation algorithms from the perspective of Equivalence Hypothesis Testing in the context of software effort estimation. The dimensionless measure MIEratio, which is independent of the units of measurements, was defined as a criterion for the assessment of evaluation models.

A probability distribution for each method was generated by grouping all MIEratios by method. These distributions answer the question: How close is the method to the best estimation? The final goal in the estimation task would be to have a perfect estimation, i.e., the absolute residual is 0. Therefore, the lower the MIEratio is the closer we are to 0. The limit for improvement lies at 0, which is the point where the estimations match the actual values and where there is no room for further improvement. The scale $(0 - \infty)$ used in the MIEratios allows a more clear identification of the differences.

According to the experimental simulations, the best intervals among all techniques, obtained by comparing the High Posterior Density intervals in the datasets used, are those of the genetic programming technique and of the linear regression with least mean squares.

Acknowledgements

Partial support has been received by Project Iceberg FP7-People-2012-IAPP-324356 (D. Rodriguez) and Project TIN2013-46928-C3 (D. Rodriguez and J.J. Dolado) and EPSRC. The authors are thankful to Mannu Satpathy for his valuable comments.

References

[1] A. Arcuri, L. Briand, A practical guide for using statistical tests to assess randomized algorithms in software engineering, in: 33^{rd} International Conference on Software Engineering (ICSE'11), ACM, New York, NY, USA, 2011, pp. 1–10.

[2] J. Derrac, S. García, D. Molina, F. Herrera, A practical tutorial on the use of nonparametric statistical tests as a methodology for comparing evolutionary and swarm intelligence algorithms, Swarm and Evolutionary Computation 1 (2011) 3–18.

[3] N. Mittas, I. Mamalikidis, L. Angelis, A framework for comparing multiple cost estimation methods using an automated visualization toolkit, Information and Software Technology 57 (2015) 310–328.

[4] C. Catal, Performance evaluation metrics for software fault prediction studies, Acta Polytechnica Hungarica 9 (4) (2012) 193–206.

[5] R. Nuzzo, Statistical errors, Nature 506 (7487) (2014) 150–152.

[6] C. Woolston, Psychology journal bans p values, Nature 519 (2015) 9.

[7] M. Shepperd, S. MacDonell, Evaluating prediction systems in software project estimation, Information and Software Technology 54 (8) (2012) 820–827.

[8] E. Stensrud, T. Foss, B. Kitchenham, I. Myrtveit, An empirical validation of the relationship between the magnitude of relative error and project size,

in: Eighth IEEE Symposium on Software Metrics (Metrics'02), 2002, pp. 3–12.

[9] T. Foss, E. Stensrud, B. Kitchenham, I. Myrtveit, A simulation study of the model evaluation criterion mmre, IEEE Transactions on Software Engineering 29 (11) (2003) 985–995.

[10] M. Meyners, Equivalence tests – a review, Food Quality and Preference 26 (2) (2012) 231–245.

[11] S.-C. Chow, J.-P. Liu, Design and Analysis of Bioavailability and Bioequivalence Studies, Chapman&Hall, 2009.

[12] D. Hauschke, V. Steinijans, I. Pigeot, Bioequivalence Studies in Drug Development. Methods and Applications, John Wiley & Sons, 2007.

[13] J. J. Dolado, M. C. Otero, M. Harman, Equivalence hypothesis testing in experimental software engineering, Software Quality Journal 22 (2) (2014) 215–238.

[14] S. Wellek, Testing statistical hypotheses of equivalence and noninferiority, 2nd Edition, Chapman & Hall, Boca Raton, Florida, USA, 2010.

[15] D. J. Schuirmann, A comparison of the two one-sided tests procedure and the power approach for assessing the equivalence of average bioavailability, Journal of Pharmacokinetics and Biopharmaceutics 15 (6) (1987) 657–680.

[16] R. L. Berger, J. C. Hsu, Bioequivalence trials, intersection-union tests and equivalence confidence sets, Statistical Science 11 (4) (1996) 283–302.

[17] W. J. Westlake, Use of confidence intervals in analysis of comparative bioavailability trials, Journal of Pharmaceutical Sciences 61 (8) (1972) 1340–1341.

[18] W. J. Westlake, Symmetrical confidence intervals for bioequivalence trials, Biometrics 32 (4) (1976) 741–744.

[19] T. B. Kirkwood, W. Westlake, Bioequivalence testing–a need to rethink, Biometrics 37 (3) (1981) 589–594.

[20] S. Lei, M. R. Smith, Evaluation of several nonparametric bootstrap methods to estimate confidence intervals for software metrics, IEEE Transactions on Software Engineering 29 (11) (2003) 996–1004.

[21] M. D. Ugarte, A. F. Militino, A. T. Arnholt, Probability and Statistics with R, CRC Press, 2008.

[22] B. Efron, Better bootstrap confidence intervals, Journal of the American Statistical Association 82 (397) (1987) 171–185.

[23] R Core Team, R: A Language and Environment for Statistical Computing, R Foundation for Statistical Computing, Vienna, Austria (2015).
URL http://www.R-project.org/

[24] A. P. Robinson, R. E. Froese, Model validation using equivalence tests, Ecological Modelling 176 (3-4) (2004) 349–358.

[25] A. P. Robinson, R. A. Duursma, J. D. Marshall, A regression-based equivalence test for model validation: shifting the burden of proof, Tree Physiology 25 (2005) 903–913.

[26] L. P. Leites, A. P. Robinson, N. L. Crookston, Accuracy and equivalence testing of crown ratio models and assessment of their impact on diameter growth and basal area increment predictions of two variants of the forest vegetation simulator, Canadian Journal of Forest Research 39 (2009) 655–665.

[27] M. Borg, D. Pfahl, Do better ir tools improve the accuracy of engineers traceability recovery?, in: Proceedings of the International Workshop on Machine Learning Technologies in Software Engineering (MALETS '11), 2011, pp. 27–34.

[28] M. Jørgensen, K. H. Teigen, K. Moløkken, Better sure than safe? overconfidence in judgement based software development effort prediction intervals, Journal of Systems and Software 70 (1) (2004) 79–93.

[29] N. Mittas, L. Angelis, Bootstrap prediction intervals for a semi-parametric software cost estimation model, in: Software Engineering and Advanced Applications, 2009. SEAA'09. 35th Euromicro Conference on, IEEE, 2009, pp. 293–299.

[30] N. Mittas, Evaluating the performances of software cost estimation models through prediction intervals, Journal of Engineering Science and Technology Review 4 (3) (2011) 266–270.

[31] M. Klas, A. Trendowicz, Y. Ishigai, H. Nakao, Handling estimation uncertainty with bootstrapping: Empirical evaluation in the context of hybrid prediction methods, in: International Symposium on Empirical Software Engineering and Measurement (ESEM 2011), IEEE, 2011, pp. 245–254.

[32] T. Heskes, Practical confidence and prediction intervals, Advances in neural information processing systems (1997) 176–182.

[33] D. R. Helsel, R. M. Hirsch, Statistical methods in water resources, Vol. 323, US Geological survey Reston, Va., 2002.

[34] R. Hyndman, G. Athanasopoulos, Forecasting: principles and practice, 2013, accessed on April, 2013.
URL http://otexts.com/fpp/

[35] G. Cumming, Understanding the New Statistics: Effect Sizes, Confidence Intervals, and Meta-Analysis, Routled, 2011.

[36] J. J. Dolado, L. Fernandez, Genetic programming, neural networks and linear regression in software project estimation, in: C. Hawkins, M. Ross, G. Staples, J. B. Thompson (Eds.), International Conference on Software Process Improvement, Research, Education and Training, British Computer Society, London, 1998, pp. 157–171.

[37] J. J. Dolado, On the problem of the software cost function, Information and Software Technology 43 (1) (2001) 61–72.

[38] J. Wen, S. Li, Z. Lin, Y. Hu, C. Huang, Systematic literature review of machine learning based software development effort estimation models, Information and Software Technology 54 (1) (2012) 41–59.

[39] T. Menzies, M. Shepperd, Special issue on repeatable results in software engineering prediction, Empirical Software Engineering 17 (1) (2012) 1–17.

[40] V. Kampenes, T. Dybå, J. Hannay, D. Sjøberg, A systematic review of effect size in software engineering experiments, Information and Software Technology 49 (11) (2007) 1073–1086.

[41] R. J. Hyndman, A. B. Koehler, Another look at measures of forecast accuracy, International Journal of Forecasting 22 (4) (2006) 679–688.

[42] B. Miranda, B. Sturtevant, J. Yang, E. Gustafson, Comparing fire spread algorithms using equivalence testing and neutral landscape models, Landscape Ecology 24 (2009) 587–598.

[43] M. Meyners, Least equivalent allowable differences in equivalence testing, Food Quality and Preference 18 (2007) 541–547.

[44] W. Langdon, J. Dolado, F. Sarro, M. Harman, Exact mean absolute error of baseline predictor marp0, Information and Software Technology, accepted for publication.

[45] I. Witten, E. Frank, M. Hall, Data Mining: Practical machine learning tools and techniques, 3rd Edition, Morgan Kaufmann, San Francisco, 2011.

[46] M. Shepperd, C. Schofield, Estimating software project effort using analogies, IEEE Transactions on Software Engineering 23 (11) (1997) 736–743.

[47] W. B. Langdon, R. Poli, Foundations of Genetic Programming, Springer, 2001.

[48] R. Poli, W. B. Langdon, N. F. Mcphee, A Field Guide to Genetic Programming, no. March, Lulu, 2008.
URL `http://www.gp-field-guide.org.uk/`

[49] L. Breiman, J. Friedman, R. Olshen, C. Stone, Classification and Regression Trees, Chapman and Hall (Wadsworth and Inc.), 1984.

[50] J. Quinlan, Learning with continuous classes, in: Proceedings of the 5th Australian Joint Conference on Artificial Intelligence, 1992, pp. 343–348.

[51] K. Maxwell, Applied Statistics for Software Managers, Software Quality Institute Series, Prentice Hall PTR, 2002.

[52] B. Kitchenham, S. L. Pfleeger, B. McColl, S. Eagan, An empirical study of maintenance and development estimation accuracy, Journal of Systems and Software 64 (1) (2002) 57–77.

[53] T. Menzies, A. Butcher, A. Marcus, T. Zimmermann, D. Cok, Local vs. global models for effort estimation and defect prediction, in: Proceedings of the 2011 26th IEEE/ACM International Conference on Automated Software Engineering, ASE'11, IEEE Computer Society, Washington, DC, USA, 2011, pp. 343–351.

[54] C. Lokan, T. Wright, P. Hill, M. Stringer, Organizational benchmarking using the isbsg data repository, IEEE Software 18 (5) (2001) 26 –32.

[55] D. Rodriguez, M. Sicilia, E. Garcia, R. Harrison, Empirical findings on team size and productivity in software development, Journal of Systems and Software 85 (3) (2012) 562–570.

[56] S. Barker, M. Shepperd, M. Aylett, The analytic hierarchy process and data-less prediction, Empirical Software Engineering Research Group ESERG: TR98-04.

[57] N. Mittas, L. Angelis, Ranking and clustering software cost estimation models through a multiple comparisons algorithm, IEEE Transactions on Software Engineering 39 (4) (2013) 537–551.

[58] P. I. Good, Resampling methods: a practical guide to data analysis, 3rd Edition, Birkhäuser, 2006.

[59] M. Shepperd, G. Kadoda, Comparing software prediction techniques using simulation, IEEE Transactions on Software Engineering 27 (11) (2001) 1014–1022.

[60] J. VanderPlas, Frequentism and bayesianism: A python-driven primer, arXiv preprint arXiv:1411.5018.

[61] E. Jaynes, O. Kempthorne, Confidence intervals vs bayesian intervals, in: W. Harper, C. Hooker (Eds.), Foundations of Probability Theory, Statistical Inference, and Statistical Theories of Science, Vol. 6b of The University of Western Ontario Series in Philosophy of Science, Springer Netherlands, 1976, pp. 175–257.

[62] M. K. Cowles, Applied Bayesian statistics: with R and OpenBUGS examples, Vol. 98, Springer Science & Business Media, 2013.

[63] R. E. Neapolitan, Learning bayesian networks, Vol. 38, Prentice Hall Upper Saddle River, 2004.

[64] S. LLC, LaplacesDemon: Complete Environment for Bayesian Inference, r package version 15.03.19 (2015).
URL http://www.bayesian-inference.com/software

Method	Dataset	$\overline{MAR}_{P_0}$	MAR	gMAR	MMRE	MdMRE	Prd(0.25)	MIE	SA	**MIEratio**
GP	CSC(fld3)	7519.422	1272.354	59.737	0.222	0.136	0.646	209.463	0.831	**0.029**
LMS	CSC(fld3)	7519.422	1336.410	275.156	0.220	0.153	0.625	407.243	0.822	**0.057**
M5P	CSC(fld3)	7519.422	1545.455	305.637	0.260	0.202	0.604	463.690	0.794	**0.066**
LR	CSC(fld3)	7519.422	1288.356	380.422	0.240	0.215	0.583	524.176	*0.829*	**0.075**
IBk	CSC(fld3)	7519.422	3102.979	380.758	0.254	0.212	0.625	583.324	0.587	**0.084**
MLP	CSC(fld3)	7519.422	2675.423	445.082	0.354	0.225	0.583	623.923	*0.644*	**0.090**
RTree	CSC(fld3)	7519.422	3394.569	437.664	0.306	0.219	0.562	676.952	0.549	**0.099**
IBk	CSC(fld2)	1315.700	274.729	141.111	0.219	0.165	0.646	189.628	*0.791*	**0.168**
LMS	CSC(fld2)	1315.700	301.627	141.407	0.236	0.155	0.646	197.306	*0.771*	**0.176**
GP	CSC(fld1)	2017.656	542.959	134.381	0.314	0.140	0.653	308.481	*0.731*	**0.180**
MLP	CSC(fld1)	2017.656	571.323	219.519	0.465	0.165	0.694	319.590	*0.717*	**0.188**
LMS	CSC(fld1)	2017.656	546.280	230.030	0.320	0.173	0.673	323.538	*0.729*	**0.191**
IBk	CSC(fld1)	2017.656	563.694	241.057	0.324	0.187	0.694	333.918	*0.721*	**0.198**
M5P	CSC(fld2)	1315.700	334.908	161.968	0.250	0.183	0.667	218.669	*0.745*	**0.199**
GP	CSC(fld2)	1315.700	322.417	126.383	0.254	0.212	0.562	221.680	*0.755*	**0.203**
M5P	CSC(fld1)	2017.656	668.263	247.102	0.320	0.159	0.694	352.094	*0.669*	**0.211**
MLP	CSC(fld2)	1315.700	326.409	175.741	0.273	0.211	0.625	231.912	*0.752*	**0.214**
LMS	ISBSG(fld3)	4721.474	2335.299	733.073	1.530	0.522	0.211	841.655	0.505	**0.217**
M5P	ISBSG(fld1)	4479.924	1751.388	703.342	1.600	0.488	0.277	801.238	*0.609*	**0.218**
GP	ISBSG(fld3)	4721.474	2385.943	719.476	0.970	0.607	0.192	846.011	0.495	**0.218**
M5P	ISBSG(fld3)	4721.474	1987.367	749.641	1.688	0.546	0.265	857.184	*0.579*	**0.222**
M5P	Maxwell(fld2)	12164.000	4134.663	1275.744	0.426	0.289	0.381	2298.224	*0.660*	**0.233**
LMS	China(fld3)	5819.186	2588.090	930.265	1.006	0.572	0.229	1132.295	*0.555*	**0.242**
LMS	ISBSG(fld1)	4479.924	2306.632	760.563	2.153	0.586	0.226	878.556	0.485	**0.244**
GP	ISBSG(fld1)	4479.924	2322.634	745.572	0.938	0.586	0.192	883.946	0.482	**0.246**
LMS	ISBSG(fld2)	3496.595	1782.202	625.202	1.280	0.478	0.240	721.919	*0.490*	**0.260**
GP	China(fld2)	4446.223	2274.440	771.991	0.743	0.609	0.223	939.076	*0.488*	**0.268**
M5P	China(fld3)	5819.186	2756.619	1056.587	1.352	0.591	0.193	1287.121	*0.526*	**0.284**
RTree	ISBSG(fld3)	4721.474	2258.100	923.286	1.927	0.581	0.208	1048.239	*0.522*	**0.285**
LMS	China(fld1)	4441.700	2267.206	836.736	1.029	0.512	0.228	1005.057	*0.490*	**0.292**

Table 2: This table shows the summary results for the best 30 values in ascending order of MIEratio ($\alpha = 0.05$).

	Qtle. 2.5%-97.5%	HPD low-upper	M-Hast. 2.5%-97.5%
GP	0.082-1.057	0.029-1.304	0.279-0.787
IBk	0.114-1.658	0.084-2.06	0.354-0.891
LMS	0.099-1.261	0.057-1.673	0.267-0.645
LR	0.147-1.409	0.075-1.577	0.409-1.005
M5P	0.112-0.806	0.066-0.908	0.275-0.585
MLP	0.12-1.207	0.09-1.356	0.367-0.971
RTree	0.16-15.242	0.099-21.263	0.895-7.867

Table 3: This table shows different probabilistic intervals for each one of the 7 methods ($\alpha = 0.05$) for the data of the MIEratios. Scale is 0-∞. Lower values are better.

	Qtle. 2.5%-97.5%	HPD low-upper	M-Hast. 2.5%-97.5%
GP	0.108-0.749	0.1-0.764	0.316-0.691
IBk	0.118-0.81	0.114-0.85	0.362-0.781
LMS	0.113-0.691	0.106-0.724	0.27-0.559
LR	0.23-3.123	0.204-3.808	0.582-1.586
M5P	0.128-0.778	0.125-0.82	0.3-0.595
MLP	0.182-1.647	0.145-1.99	0.461-0.978
RTree	0.334-1.962	0.329-2.022	0.775-1.837

Table 4: This table shows different probabilistic intervals for each one of the 7 methods ($\alpha = 0.05$) for the means of the MIEratios in 10 runs. Scale is 0-∞. Lower values are better.

	Qtle. 2.5%-97.5%	HPD low-upper	M-Hast. 2.5%-97.5%
GP	0.289-0.804	0.217-0.831	0.47-0.672
IBk	0.307-0.766	0.297-0.791	0.449-0.597
LMS	0.417-0.804	0.38-0.822	0.519-0.643
LR	0.29-0.763	0.258-0.829	0.445-0.61
M5P	0.268-0.777	0.192-0.794	0.485-0.698
MLP	0.138-0.741	0.104-0.752	0.345-0.681
RTree	0.235-0.541	0.225-0.549	0.333-0.471

Table 5: This table shows different credible intervals for each one of the 7 methods ($\alpha = 0.05$) for the data of the SA. Scale is 0-1. Greater values are better.

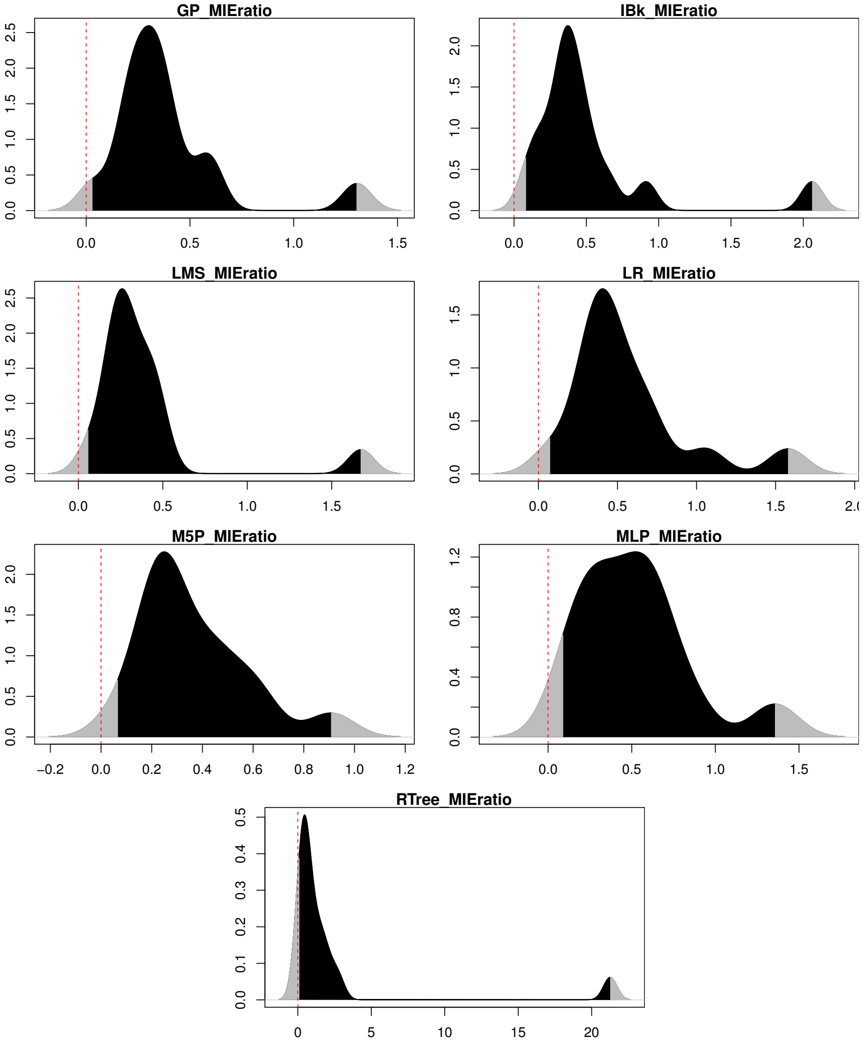


Figure 7: High Posterior Density intervals of the MIEratios of the Methods